Differential geometry began as the study of curves and surfaces using the methods of calculus. In time, the notions of curve and surface were generalized along with associated notions such as length, volume, and curvature. At the same time the topic has become closely allied with developments in topology. The basic object is a smooth manifold, to which some extra structure has been attached, such as a Riemannian metric, a symplectic form, a distinguished group of symmetries, or a connection on the tangent bundle. This book is a graduate-level introduction to the tools and structures of modern differential geometry. Included are the topics usually found in a course on differentiable manifolds, such as vector bundles, tensors, differential forms, de Rham cohomology, the Frobenius theorem and basic Lie group theory. The book also contains material on the general theory of connections on vector bundles and an in-depth chapter on semi-Riemannian geometry that covers basic material about Riemannian manifolds and Lorentz manifolds. An unusual feature of the book is the inclusion of an early chapter on the differential geometry of hyper-surfaces in Euclidean space. There is also a section that derives the exterior calculus version of Maxwell's equations. The first chapters of the book are suitable for a one-semester course on manifolds. There is more than enough material for a year-long course on manifolds and geometry.
Differential geometry arguably offers the smoothest transition from the standard university mathematics sequence of the first four semesters in calculus, linear algebra, and differential equations to the higher levels of abstraction and proof encountered at the upper division by mathematics majors. Today it is possible to describe differential geometry as "the study of structures on the tangent space," and this text develops this point of view. This book, unlike other introductory texts in differential geometry, develops the architecture necessary to introduce symplectic and contact geometry alongside its Riemannian cousin. The main goal of this book is to bring the undergraduate student who already has a solid foundation in the standard mathematics curriculum into contact with the beauty of higher mathematics. In particular, the presentation here emphasizes the consequences of a definition and the careful use of examples and constructions in order to explore those consequences.

Differential Forms and Applications

This monograph is the first one to systematically present a series of local and global estimates and inequalities for differential forms, in particular the ones that satisfy the A-harmonic equations. The presentation focuses on the Hardy-Littlewood, Poincare, Caccioppoli, imbedded and reverse Holder inequalities. Integral estimates for operators, such as homotopy operator, the Laplace-Beltrami operator, and the gradient operator are discussed next. Additionally, some related topics such as BMO inequalities, Lipschitz classes, Orlicz spaces and inequalities in Carnot groups are discussed in the concluding chapter. An abundance of bibliographical references and historical material supplement the text throughout. This rigorous presentation requires a familiarity with topics such as differential forms, topology and Sobolev space theory. It will serve as an invaluable reference for researchers, instructors and graduate students in analysis and partial differential equations and could be used as additional material for specific courses in these fields.

Differential Geometry of Curves and Surfaces

An explanation of the mathematics needed as a foundation for a deep understanding of general relativity or quantum field theory. Physics is naturally expressed in mathematical language. Students new to the subject must simultaneously learn an idiomatic mathematical language and the content that is expressed in that language. It is as if they were asked to read Les Misérables while struggling with French grammar. This book offers an innovative way to learn the differential geometry needed as a foundation for a deep understanding of general relativity or quantum field theory as taught at the college level. The approach taken by the authors (and used in their classes at MIT for many years) differs from the conventional one in several ways, including an emphasis on the development of the covariant derivative and an avoidance of the use of traditional index notation for tensors in favor of a semantically richer language of vector fields and differential forms. But the biggest single difference is the authors' integration of computer programming into their explanations. By programming a computer to interpret a formula, the student soon learns whether or not a formula is correct. Students are led to improve their program, and as a result improve their understanding.

Applied Differential Geometry

Multivariable Mathematics combines linear algebra and multivariable mathematics in a rigorous approach. The material is integrated to emphasize the recurring theme of implicit...
This monograph provides an introduction to, as well as a unification and extension of the published work and some unpublished ideas of J. Lipman and E. Kunz about traces of differential forms and their relations to duality theory for projective morphisms. The approach uses Hochschild-homology, the definition of which is extended to the category of topological algebras. Many results for Hochschild-homology of commutative algebras also hold for Hochschild-homology of topological algebras. In particular, after introducing an appropriate notion of completion of differential algebras, one gets a natural transformation between differential forms and Hochschild-homology of topological algebras. Traces of differential forms are of interest to everyone working with duality theory and residue symbols. Hochschild-homology is a useful tool in many areas of $k$-theory. The treatment is fairly elementary and requires only little knowledge in commutative algebra and algebraic geometry.

Traces of Differential Forms and Hochschild Homology

This text presents a graduate-level introduction to differential geometry for mathematics and physics students. The exposition follows the historical development of the concepts of connection and curvature with the goal of explaining the Chern–Weil theory of characteristic classes on a principal bundle. Along the way we encounter some of the high points in the history of differential geometry, for example, Gauss' Theorema Egregium and the Gauss–Bonnet theorem. Exercises throughout the book test the reader's understanding of the material and sometimes illustrate extensions of the theory. Initially, the prerequisites for the reader include a passing familiarity with manifolds. After the first chapter, it becomes necessary to understand and manipulate differential forms. A knowledge of de Rham cohomology is required for the last third of the text. Prerequisite material is contained in the author's text An Introduction to Manifolds, and can be learned in one semester. For the benefit of the reader and to establish common notations, Appendix A recalls the basics of manifold theory. Additionally, in an attempt to make the exposition more self-contained, sections on algebraic constructions such as the tensor product and the exterior power are included. Differential geometry, as its name implies, is the study of geometry using differential calculus. It dates back to Newton and Leibniz in the seventeenth century, but it was not until the nineteenth century, with the work of Gauss on surfaces and Riemann on the curvature tensor, that differential geometry flourished and its modern foundation was laid. Over the past one hundred years, differential geometry has proven indispensable to an understanding of the physical world, in Einstein's general theory of relativity, in the theory of gravitation, in gauge theory, and now in string theory. Differential geometry is also useful in topology, several complex variables, algebraic geometry, complex manifolds, and dynamical systems, among other fields. The field has even found applications to group theory as in Gromov's work and to probability theory as in Diaconis's work. It is not too far-fetched to argue that differential geometry should be in every mathematician's arsenal.
Read Book Differential Forms And The Geometry Of General Relativity

undergraduate or beginning graduate students in mathematics or physics, most of the text requires little more than familiarity with calculus and linear algebra. The first half presents an introduction to general relativity that describes some of the surprising implications of relativity without introducing more formalism than necessary. This nonstandard approach uses differential forms rather than tensor calculus and minimizes the use of "index gymnastics" as much as possible. The second half of the book takes a more detailed look at the mathematics of differential forms. It covers the theory behind the mathematics used in the first half by emphasizing a conceptual understanding instead of formal proofs. The book provides a language to describe curvature, the key geometric idea in general relativity.

Holomorphic Functions and Integral Representations in Several Complex Variables

This edition of the invaluable text Modern Differential Geometry for Physicists contains an additional chapter that introduces some of the basic ideas of general topology needed in differential geometry. A number of small corrections and additions have also been made. These lecture notes are the content of an introductory course on modern, coordinate-free differential geometry which is taken by first-year theoretical physics PhD students, or by students attending the one-year MSc course "Quantum Fields and Fundamental Forces" at Imperial College. The book is concerned entirely with mathematics proper, although the emphasis and detailed topics have been chosen bearing in mind the way in which differential geometry is applied these days to modern theoretical physics. This includes not only the traditional area of general relativity but also the theory of Yang-Mills fields, nonlinear sigma models and other types of nonlinear field systems that feature in modern quantum field theory. The volume is divided into four parts: (i) introduction to general topology; (ii) introductory coordinate-free differential geometry; (iii) geometrical aspects of the theory of Lie groups and Lie group actions on manifolds; (iv) introduction to the theory of fibre bundles. In the introduction to differential geometry the author lays considerable stress on the basic ideas of "tangent space structure", which he develops from several different points of view — some geometrical, others more algebraic. This is done with awareness of the difficulty which physics graduate students often experience when being exposed for the first time to the rather abstract ideas of differential geometry.

A New Approach to Differential Geometry using Clifford's Geometric Algebra

This is a self-contained introductory textbook on the calculus of differential forms and modern differential geometry. The intended audience is physicists, so the author emphasises applications and geometrical reasoning in order to give results and concepts a precise but intuitive meaning without getting bogged down in analysis. The large number of diagrams helps elucidate the fundamental ideas. Mathematical topics covered include differentiable manifolds, differential forms and twisted forms, the Hodge star operator, exterior differential systems and symplectic geometry. All of the mathematics is motivated and illustrated by useful physical examples.

Differential Geometry

This book provides an introduction to the basic concepts in differential topology, differential geometry, and differential equations, and some of the main basic theorems in all three areas. This new edition includes new chapters, sections, examples, and exercises. From the reviews:
There are many books on the fundamentals of differential geometry, but this one is quite exceptional; this is not surprising for those who know Serge Lang's books.
Differential Forms and the Geometry of General Relativity

Symplectic and Poisson geometry emphasizes group actions, momentum mappings, and reductions. This book gives the careful reader working knowledge in a wide range of topics of modern coordinate-free differential geometry in not too many pages. A prerequisite for using this book is a good knowledge of undergraduate analysis and linear algebra.

Multivariable Mathematics

An application of differential forms for the study of some local and global aspects of the differential geometry of surfaces. Differential forms are introduced in a simple way that will make them attractive to "users" of mathematics. A brief and elementary introduction to differentiable manifolds is given so that the main theorem, namely Stokes' theorem, can be presented in its natural setting. The applications consist in developing the method of moving frames expounded by E. Cartan to study the local differential geometry of immersed surfaces in $\mathbb{R}^3$ as well as the intrinsic geometry of surfaces. This is then collated in the last chapter to present Chern's proof of the Gauss-Bonnet theorem for compact surfaces.

Cohomology and Differential Forms

This is a book that the author wishes had been available to him when he was student. It reflects his interest in knowing (like expert mathematicians) the most relevant mathematics for theoretical physics, but in the style of physicists. This means that one is not facing the study of a collection of definitions, remarks, theorems, corollaries, lemmas, etc. but a narrative — almost like a story being told — that does not impede sophistication and deep results. It covers differential geometry far beyond what general relativists perceive they need to know. And it introduces readers to other areas of mathematics that are of interest to physicists and mathematicians, but are largely overlooked. Among these is Clifford Algebra and its uses in conjunction with differential forms and moving frames. It opens new research vistas that expand the subject matter. In an appendix on the classical theory of curves and surfaces, the author slashes not only the main proofs of the traditional approach, which uses vector calculus, but even existing treatments that also use differential forms for the same purpose.

Contents:

- Introduction: Orientations
- Tools: Differential Forms
- Vector Spaces and Tensor Products
- Exterior Differentiation
- Two Klein Geometries: Affine Klein Geometry
- Euclidean Klein Geometry
- Cartan Connections: Generalized Geometry Made Simple
- Affine Connections
- Euclidean Connections
- Riemannian Spaces and Pseudo-Spaces
- The Future?: Extensions of Cartan
- Understand the Past to Imagine the Future
- A Book of Farewells

Readership: Physicists and mathematicians working on differential geometry.

Keywords: Differential Geometry; Differential Forms; Moving Frames; Exterior Calculus

Key Features: Reader Friendly, Naturalness, Respect for the history of the subject and related incisiveness

Fundamentals of Differential Geometry

An important question in geometry and analysis is to know when two k-forms $f$ and $g$ are equivalent through a change of variables. The problem is therefore to find a map $\phi$ so that it satisfies the pullback equation: $\phi^*(g) = f$. In more physical terms, the question under consideration can be seen as a problem of mass transportation. The problem has received considerable attention in the cases $k = 2$ and $k = n$, but much less when $3 \leq k \leq n-1$. The present monograph provides the first comprehensive study of the equation. The work begins by recounting various properties of exterior forms and differential forms that prove
Visual Differential Geometry and Forms

Since the times of Gauss, Riemann, and Poincare, one of the principal goals of the study of manifolds has been to relate local analytic properties of a manifold with its global topological properties. Among the high points on this route are the Gauss-Bonnet formula, the de Rham complex, and the Hodge theorem; these results show, in particular, that the central tool in reaching the main goal of global analysis is the theory of differential forms. This book is a comprehensive introduction to differential forms. It begins with a quick presentation of the notion of differentiable manifolds and then develops basic properties of differential forms as well as fundamental results about them, such as the de Rham and Frobenius theorems. The second half of the book is devoted to more advanced material, including Laplacians and harmonic forms on manifolds, the concepts of vector bundles and fiber bundles, and the theory of characteristic classes. Among the less traditional topics treated in the book is a detailed description of the Chern-Weil theory. With minimal prerequisites, the book can serve as a textbook for an advanced undergraduate or a graduate course in differential geometry.
Read Book Differential Forms And The Geometry Of General Relativity

This book explains and helps readers to develop geometric intuition as it relates to differential forms. It includes over 250 figures to aid understanding and enable readers to visualize the concepts being discussed. The author gradually builds up to the basic ideas and concepts so that definitions, when made, do not appear out of nowhere, and both the importance and role that theorems play is evident as or before they are presented. With a clear writing style and easy-to-understand motivations for each topic, this book is primarily aimed at second- or third-year undergraduate math and physics students with a basic knowledge of vector calculus and linear algebra.

The Pullback Equation for Differential Forms

This volume presents a collection of problems and solutions in differential geometry with applications. Both introductory and advanced topics are introduced in an easy-to-digest manner, with the materials of the volume being self-contained. In particular, curves, surfaces, Riemannian and pseudo-Riemannian manifolds, Hodge duality operator, vector fields and Lie series, differential forms, matrix-valued differential forms, Maurer–Cartan form, and the Lie derivative are covered. Readers will find useful applications to special and general relativity, Yang–Mills theory, hydrodynamics and field theory. Besides the solved problems, each chapter contains stimulating supplementary problems and software implementations are also included. The volume will not only benefit students in mathematics, applied mathematics and theoretical physics, but also researchers in the field of differential geometry.

Manifolds and Differential Geometry

An inviting, intuitive, and visual exploration of differential geometry and forms Visual Differential Geometry and Forms fulfills two principal goals. In the first four acts, Tristan Needham puts the geometry back into differential geometry. Using 235 hand-drawn diagrams, Needham deploys Newton's geometrical methods to provide geometrical explanations of the classical results. In the fifth act, he offers the first undergraduate introduction to differential forms that treats advanced topics in an intuitive and geometrical manner. Unique features of the first four acts include: four distinct geometrical proofs of the fundamentally important Global Gauss-Bonnet theorem, providing a stunning link between local geometry and global topology; a simple, geometrical proof of Gauss's famous Theorema Egregium; a complete geometrical treatment of the Riemann curvature tensor of an n-manifold; and a detailed geometrical treatment of Einstein's field equation, describing gravity as curved spacetime (General Relativity), together with its implications for gravitational waves, black holes, and cosmology. The final act elucidates such topics as the unification of all the integral theorems of vector calculus; the elegant reformulation of Maxwell's equations of electromagnetism in terms of 2-forms; de Rham cohomology; differential geometry via Cartan's method of moving frames; and the calculation of the Riemann tensor using curvature 2-forms. Six of the seven chapters of Act V can be read completely independently from the rest of the book. Requiring only basic calculus and geometry, Visual Differential Geometry and Forms provocatively rethinks the way this important area of mathematics should be considered and taught.

Copyright code: 74bef83e793402272d238b2cb8a43380
Copyright: learning.acttraining.org.uk

Page 12/12